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Abstract

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Finding the Shortest Path in Huge Data Traffic Networks: a Hybrid Speed Model

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***Abstract*—The shortest path problem has become an important issue in the increasingly complex road networks nowadays, especially for these applications which are strict with high timeliness. However, searching the shortest path is difficult as road traffic flows are time-varying. An important issue in searching the shortest path is how to obtain the time expired on each segment at the given time. To this purpose, we propose a hybrid speed model to calculate the travel time in this paper, which considers the difference between the speed in congested and uncongested road networks. And analysis of speed in both conditions are given, respectively. Subsequently, a metric is also proposed to distinguish between congested and uncongested networks. Our work also utilizes the huge traffic data to reflect and analyze the real scenario. Compared to the previous work, this paper considers a more complex urban traffic scenario and some verifications are made with our data. Finally, a numerical study is carried out in the urban road network in Kaohsiung, Taiwan. The results show the hybrid speed model can give travel time prediction in an accurate way and can provide useful information for road designers.**

I. INTRODUCTION

Transportation system has played a vitally important role in the rapid development of the society as it is fundamental to people's lives and the growth of the economics. However, modern cities have seen a severe traffic problem in recent years that the traffic networks and the urban planning cannot keep up with the continuously increasing number of vehicles. This has posed a great challenge to our traffic designers and scientific researchers.

With deterioration of traffic condition, people spend much more time on road than ever before, which is a huge waste. The problem of travel time in road networks was first modeled as the Vehicle Routing Problem (VRP) [1]–[3]. The VRP is proposed as the generalization of the Traveling Salesman Problem (TSP) presented by Flood [2]. The objective of VRP is to find a set of routes for the vehicles, covering all the nodes (destinations that must be served) and making the total length travelled minimal. Different from the VRP problem, the shortest path finding problem focuses on providing the paths with shortest time for drivers, which is considered from an individual perspective. Although the shortest path has no consideration for overall performance, it is of great value in these applications requiring extremely high timeliness such as the emergency rescue.

The shortest path finding problem was first studied as a static problem. The time cost on each road segment is defined as a linear function of the segment's length. It is very visualized and it is easy for computation. However, this model is not fit for the urban road networks as the urban traffic flows fluctuate greatly in a day. Therefore, a lot of works focused on dynamic shortest path have been made. Most of them are in the field of model construction, with which the researchers want to reflect the real traffic information (e.g., traffic flows fluctuation and speed variance). The time cost calculation in the literature generally use the speed-based travel time estimation model. Almost all these works can be divided into two categories. Most of them pay attention to the freeway scenario, where the vehicle speed changes in a very narrow range. The rest of them focus on the urban road scenario, where the traffic is always congested. There are few works about the overall analysis of the situations in urban road networks. In fact, using only one model in the urban scenario is not appropriate. According to the traffic flow fluctuation presented by our traffic data, the traffic condition of urban roads is the combination of the both. Moreover, the traffic data are always used to calculate the link travel time rather than to verify the speed model. Therefore, more works are needed for the verification of the proposed speed model.

To solve the problems above, we propose a hybrid speed model in this paper. The uncongested model is given to characterize the properties of speed in uncongested traffic condition. The congested model is also proposed and analyzed with the consideration of congested traffic conditions. The hybrid model covers both uncongested and congested traffic conditions, which is an overall and truthful reflection of the urban road networks. We also present a method to distinguish between two conditions. In order to verify the proposed model, huge traffic data are used. The hybrid model improves the accuracy of the link travel time calculation. The shortest path search algorithm using the hybrid speed model in our numerical experiment achieves a good performance.

The rest of the paper is organized as follows: In Section II, we introduce the related works of shortest path finding problem. In Section III, the hybrid speed model is proposed

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and the estimation process of link travel time is analyzed. The corresponding shortest path searching algorithm is presented at the end of this section. The characteristics of traffic data are analyzed in Section IV. In Section V, numerical experiments are carried out and corresponding analysis are given. Finally we will draw the conclusion based on the analysis and experiment results.

II. Related Works

The research of the shortest path finding problem in road networks can be divided into two parts. One is the shortest path search algorithm in topology that is widely used in numerous kinds of networks. And the other is the link travel time estimation, which plays an important role in real road networks. The predicted link travel time is the input of the search algorithm, thus its accuracy and validity have great influence on the search performance. In this part, we first introduce several works about the shortest path algorithm and then the works in the dynamic road networks

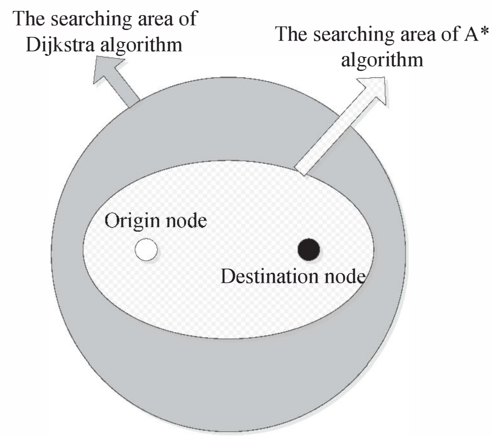

Fig. 1: Searching area of Dijkstra algorithm vs. searching area of A^* algorithm.

A. Shortest Path Algorithms

Most of the traditional shortest path algorithms belong to dynamic programming, and the optimal shortest path is identified via a recursive decision-making process. The Dijkstra algorithm [4] first proposed by Dijkstra in 1959 is one of the best known Shortest Path (SP) algorithms. It searches from source node and gradually expands its searching area to neighboring links. The Dijkstra algorithm is widely used in the research of the SP problem. However, it is not efficient in the real traffic applications like vehicle navigation due to its complexity of $O(n^2)$ which is relatively costly to the real-time searching problem. In each recursion of Dijkstra algorithm, all potential vertices that have not joined in the solution set will be considered. Obviously, the researching for some area is not needed as Fig. 1 shows. In order to reduce the unnecessary researching areas, several heuristic algorithms are proposed. Guzolek and Koch [5] discussed how heuristic search methods could be used in a vehicle navigation system. T. Kuznetsov [6] analyzed the applications of A^* algorithm, a bi-directional search method, and a hierarchical search method used for finding paths. From then on, a lot of works have been made to develop a comprehensive strategy for improving the efficiency of the shortest path search process. In this paper, we adopt the heuristic A^* algorithm for the numerical experiment, where heuristic function is provided.

B. Dynamic Road Networks

The data collected from the real traffic scenario are always used to calculate the link travel time at given time t. The link travel time of space time transport network is represented by random variable whose probability distribution functions vary over time in [7]. W. Tu et al. [8] drew the time-varying road networks with the data of floating vehicles reporting pairs of time and position information. They modeled the vehicle speed as a function of road density and maximum traveling speed at given time t. The speed is then used to calculate the link travel time of the specific road segment. Although they considered the influence of congested situation and used a congestion index to control the speed function, it is not accurate due to user-defined parameters and metrics. B. Y. Chen et al. [9] studied the speed model under the restriction of Stochastic Time-Dependent (STD) road networks. They proved that the link travel time in STD networks satisfies the Stochastic First-In-First-Out (S-FIFO) property. And they utilized the speed model in congested traffic scenario. They also provided the algorithm to calculate the link travel time and search for the shortest path. However, their model and analysis are based on the periodical real-time data collected from the moving vehicles, which is costly to realize and will be affected by the delay of the data collection process. What's more, it is restricted to a narrow scope of congested scenario. S. Kim et al. [10] developed the decision-making procedures for determining the optimal driver attendance time, optimal departure times, and optimal routing policies under time-varying traffic flows based on a Markov decision process formulation. They proved that their algorithm performs well, however, periodically updating the database of traffic information is also needed.

In our study, both the congested and uncongested road conditions are taken into consideration to build the speed model, which is in favor of the link travel time estimation. Moreover, the huge historical traffic data are used for the verification of proposed model and real-world use. With the heuristic A^* algorithm, we obtain the least expected time path with little computational complexity. Any request for the shortest path will be replied by control center in time.

III. Problem Formulation

In this section, we first propose the hybrid speed model. A detailed analysis of the link travel time estimation is contained. For the purpose of distinguishing between congested and uncongested condition, we provide the method utilizing the traffic data, subsequently. At last, heuristic A^* algorithm is adopted for the numerical experiment discussed in the next section.

A. Hybrid Speed Model

The shortest path finding problem has generally been studied from the perspective of graph theory. The road networks can be modeled as a directed graph $G = \{V, E, C\}$. In the graph G, $V = \{v_1, ..., v_n\}$ is a set of vertices and $E = \{(v_i, v_j)|(v_i, v_j) \in V^2, i \neq j\}$ is the arc set. For concise expression, we use e_{ij} to represent the arc (v_i, v_j). $C = \{c_{ij}|(v_i, v_j) \in E\}$ is the cost set of the arcs. We define T as the total time cost when traveling along the links in the road networks. T_{ij} is used to replace the c_{ij}. T can be figured out as

$$T = \sum_{e_{ij} \in E} T_{ij}\delta_{ij}, \tag{1}$$

where δ_{ij} is the incidence variable, $\delta_{ij} = 1$ means that the link e_{ij} is on the path P^{sd} from source node s to destination node d and $\delta_{ij} = 0$, otherwise. Therefore, the problem of finding the path with least expected time in a road network can be defined as

$$\min E(T) = \min E\left[T = \sum_{e_{ij} \in E} T_{ij}\delta_{ij}\right] \tag{2}$$

s.t.

$$\delta_{ij} = \begin{cases} 1 & e_{ij} \in P^{sd} \\ 0 & e_{ij} \notin P^{sd}, \end{cases} \quad i, j \in 1, 2, ..., n, i \neq j. \tag{3}$$

Different from the previous work, this paper focuses on the urban road networks with the fact that two kinds of conditions exist in road, uncongested condition and congested condition, respectively. During the uncongeseted, incident free conditions, the travel speed on the link may vary very little. While in the congested conditions, the travel speed may vary dramatically. Let $V_{ij}(t)$ be the stochastic variable representing the speed on the link e_{ij} at given time t. We discuss the speed models under the two conditions in the following part, which are named uncongested model and congested model, respectively.

1) The Uncongested Model: In the uncongested road situation, we calculate the link travel time using the time slice model. The time slice model tries to account for the variation in speed on different links as the speed is dependent on the time entering into the link. The equation for the time is listed as:

$$T_{ij}(k) = \frac{n * length(e_{ij})}{v_{ij}(1, t_1) + v_{ij}(2, t_2) + ... + v_{ij}(n, t_n)}, \tag{4}$$

where the average speed $\frac{1}{n} * \left(v_{ij}(1, t_1) + v_{ij}(2, t_2) + ... + v_{ij}(n, t_n)\right)$ on the link e_{ij} is used. n is the number of detectors deployed along the link. $v_{ij}(m, t_m)$ is the instantaneous speed at time t_m. Assuming that the entering time into e_{ij} is t_0, the time t_m can be obtained by

$$t_m = t_{m-1} + \frac{|x_m - x_{m-1}|}{2 * v_{ij}(m-1, t_{m-1})} \quad m \in \{1, 2, ..., n\}. \tag{5}$$

Each speed can be obtained by recursively calling Eq. (5).

The uncongested, incident free condition is always analyzed in the freeway situation. However, from the traffic data obtained, we find that the situation on roads is similar to the freeway situation in some periods in a day, when there are few traffic flows and speed can be maintained in a approximately fixed value. The accuracy of Eq. (4) is determined by the road segment length and the number of detectors deployed on the segment. With shorter segment and larger number, the prediction link travel time will be more accurate.

2) The Congested Model: As studied in [3], [11], the travel speed in congested condition is approximately subject to the normal distribution due to the stochastic nature of congested road networks. Assuming that there is a vehicle running on link e_{ij}, the parameters of the normal distribution can be obtained by sampling the vehicle speed. Different from this assumption, our detectors deployed along the link e_{ij} upload the speed information, which is averaged among the vehicles passing by. We can find that in a short period and on the same link, the travel speed of all vehicles experiences the majority of the speed states. In other words, it is ergodic. The speed of all the vehicles is used to represent the possible speed of a specific vehicle. Therefore, we can model the speed utilizing all vehicles' speed rather than the samples of a specific one. Let V_{ij} be subject to $N(\mu, \delta^2)$, where μ and δ^2 are the mean and variance of the normal distribution, respectively. We have

$$\mu = \frac{1}{N_\alpha} \times \sum_{i \in \alpha} v_i(t), \tag{6}$$

where α is the index set of the speed participated in calculation. N_α is the number of elements in index set and $v_i(t)$ is the uploaded speed at given time t. Having the average speed μ, the variance δ^2 can be obtain using

$$\delta^2 = \frac{1}{N_\alpha} \times \sum_{i \in \alpha} (v_i(t) - \mu)^2. \tag{7}$$

The link e_{ij} can be divided into N parts $\Delta = (\Delta L_1, \Delta L_2, ..., \Delta L_N)$, the travel speed in ΔL_k is the average value $V_{ij}(k)$. Therefore, the approximate travel time is

$$T'_{ij} = \sum_{k=1}^{N} \frac{\Delta L_k}{V_{ij}(k)}. \tag{8}$$

When the divided part is small enough, which means that the element with maximum value in partition set is subject to

$$\max\{\Delta L_1, \Delta L_2, ..., \Delta L_N\} \to 0, \tag{9}$$

the travel time consumed on link e_{ij} is calculated by

$$T_{ij} = \lim_{N\to\infty} \sum_{k=1}^{N} \frac{\Delta L_k}{V_{ij}(k)}. \tag{10}$$

Let X_{ij} be the reciprocal of the V_{ij}, Eq. (10) can be presented as

$$T_{ij} = \lim_{N\to\infty} \sum_{k=1}^{N} \Delta L_k \times X_{ij}(k). \tag{11}$$

If the link e_{ij} is split equally in terms of length, which means that $\Delta L_1 = \Delta L_2 = ... = \Delta L_N = len(\Delta)/N = dl$, Eq. (11) can be converted into

$$\begin{aligned} T_{ij} &= \lim_{N\to\infty} \sum_{k=1}^{N} \frac{len(\Delta)}{N} \times X_{ij}(k) \\ &= \int_0^{len(\Delta)} X_{ij} dl. \end{aligned} \tag{12}$$

From the distribution of V_{ij}, the Probability Density Function (PDF) of X_{ij} can be obtain as follows:

$$f_{X_{ij}}(x) = \frac{\frac{1}{x} - \mu}{\sqrt{2\pi}\delta^3 x^2} \exp\left(-\frac{(\frac{1}{x}-\mu)^2}{2\delta^2}\right). \tag{13}$$

We calculate the expectations of Eq. (12), and we can get the following form:

$$\begin{aligned} E[T_{ij}] &= E\left[\int_0^{len(\Delta)} X_{ij} dl\right] \\ &= \int_0^{len(\Delta)} E\left[X_{ij}\right] dl \\ &= E\left[X_{ij}\right] * len(\Delta). \end{aligned} \tag{14}$$

Because the speed on congested road has its limitations, mainly referring to limitations of the minimum and maximum speed. An amendment has been made to the PDF. Assuming that the minimum speed is $m(m \neq 0)$ and the maximum speed is $M(M \ll \infty)$, the PDF of X_{ij} should be revised to the following form:

$$f'_{X_{ij}}(x) = \frac{f_{X_{ij}}(x)}{\int_{1/M}^{1/m} f_{X_{ij}}(x)dx}. \tag{15}$$

From the above two speed models, we can get the link travel time in urban traffic scenarios, where both congested and uncongested road conditions exist. In order to tell the two conditions apart, a metric based on the the speed variance is given in the next subsection.

B. Conditions Identification

The parameters such as lane occupied rate can be used to tell the approximate traffic conditions. However, we will take advantage of the detected speed information to distinguish the traffic conditions, which mainly focuses on the speed fluctuation intensity. We define the speed fluctuation function as follows:

Algorithm 1 Find The Least Expected Time Path

Input: Source and destination nodes and departure time
Output: The least expected time path

function MAIN(time, speed)
  Put Source node to close set and Let it be the previous node
  Call LINKTRAVELTIMEESTIMATION(time, speed)
  Update E(i), H(i) and choose the neighbor node that has the minimum value of F(i) as the previous node and put it to the close set
end function
function LINKTRAVELTIMEESTIMATION(time, speed)
  Call CONDITIONDISTINGUISH(time, speed)
  if state = congested **then**
    Calculate the link travel time using congested model
  else
    Calculate the link travel time using uncongested model
  end if
end function
function CONDITIONDISTINGUISH(time, speed)
  Use Fluc(t) to distinguish the road conditions
end function

$$Fluc(t) = \frac{E[|v(t) - v_{avg}(t)|]}{v_{avg}(t)}, \tag{16}$$

where $E[|v(t) - v_{avg}(t)|]$ reflects the speed variation at given time t. When $Fluc(t)$ is larger than the threshold, we should adopt the congested model to calculate the link travel time. Otherwise, we should adopt the uncongested model.

C. Algorithm For Finding The Least Expected Time Path

In order to find out the least expected time path, we adopt the A^* algorithm, a classic heuristic algorithm among the shortest path algorithms. The most important part of A^* algorithm is the decision function which is presented as follows:

$$F(i) = E(i) + H(i), \tag{17}$$

where $F(i)$ is the estimated cost function of the potential travel path from the source node s to the intermediate node i. $E(i)$ is the real cost form the node s to the node d, and $H(i)$ is the estimated cost from the node i to the node d. A simple and admissible heuristic function is shown below:

$$H(i) = \frac{d_{id}}{v_{Max}}, \tag{18}$$

where the d_{id} is the Manhattan distance from node i to node d and v_{Max} is the maximum speed along the path the vehicle has traveled.

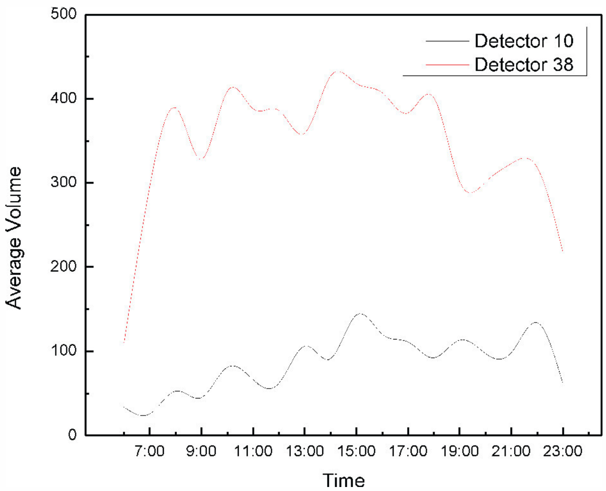

Fig. 2: Fluctuation of the traffic flows in a day.

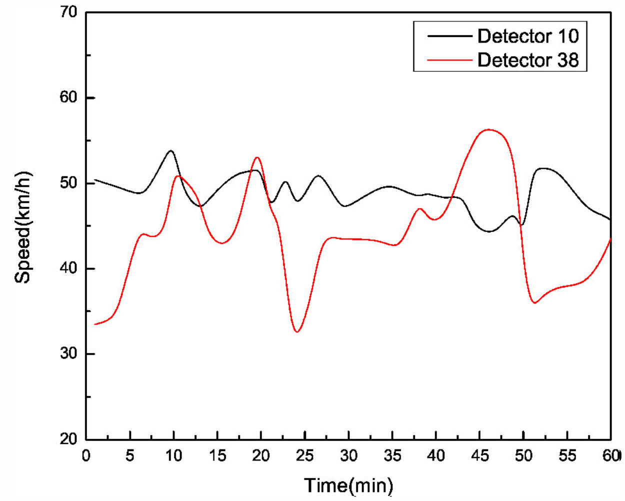

Fig. 3: The fluctuation of speed in an hour.

IV. Characteristics of the Data

A. Data Consistency

The historical data collected by the vehicle detectors can predict current traffic flow only if the data are of consistency. It means the data collected in the same period everyday are consistent in terms of some criteria. Otherwise, the prediction will be of much uncertainty, which does a lot of harm to the accuracy. Therefore, the data consistency must be verified. In this paper, some general criteria are recommended including variance and standard deviation.

B. Properties of The Traffic in a Day

In common sense, the traffic flows are not smooth in a day. Fluctuation is found greatly affected by the rush hour in the workday. The study of the variation of the traffic flow in a day can help us make a more accurate and adaptive decision. At the same time, we will obtain some important recognition of the traffic according to the topology theory. In Fig. 2, we can find that the traffic flows are different at the detector 10 and detector 38. Meanwhile the traffic flows vary in a day.

C. Speed Fluctuation

In our study, there are two kinds of road conditions existed in the same time. One of the conditions is that the speed varies smoothly, while the other is the opposite. Fig. 3 is the record of the average speed in 60 minutes from 12:00 to 13:00, which is recorded by the detector 10 and 38, respectively. We can find that in Fig. 3, the speed detected at the detector 10 has a comparatively smooth variation to that at the detector 38, which proves that it is appropriate to use the hybrid model to characterize the speed in the urban traffic scenario.

V. Numerical Experiment and Analysis

A. Case Study

We have Vehicle Detector (VD) data in hand, which are collected by the vehicle detectors deployed in the majority of the roads in Kaohsiung, Taiwan (see Fig 4). VD is consisted of the basic information of the road including the road type and its volume, and the vehicle information including the car number, car types, lane occupied rate and the average speed which are shown in Fig. 5. The VD information is updated every minute, which well reflects the information of real traffic conditions.

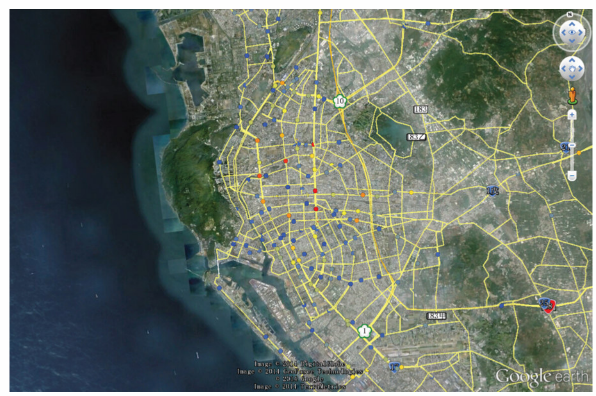

Fig. 4: The detectors deployment in Kaohsiung.


```
- <Info vdid="64000V000241" status="0" datacollecttime="2013/04/01 12:00:00">
  - <lane vsrdir="0" vsrid="0" speed="64" laneoccupy="3">
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  </Info>
```


Fig. 5: A piece of record uploaded from the detector.

B. Analysis

We choose the source and destination nodes, which are represented by the location A and B, respectively. Meanwhile

TABLE I: The Least Expected Travel Time from A to B at Different time

Time	Least expected travel time	Path
7:00	4.4 (min)	1
12:00	4.3 (min)	1
15:00	5.9 (min)	2
17:00	5.8 (min)	2
19:00	6.6 (min)	2

Fig. 6: The Shortest Path from A to B.

we carry out the experiment at different time in a day. Through the A^* algorithm based on hybrid speed model, two paths shown in Fig. 6 are chosen to be the shortest path in turns. Table I lists the least expected travel time of the paths at different time. We can find that when there is little traffic at 7:00 and 12:00 (see Fig. 2), path 2 is the better choice. While path 1 is better when there is heavy traffic at 15:00, 17:00 and 19:00. It indicates that path 1 can tolerate more traffic variance with less increment on time cost , compared to the path 2. Moreover, the results provide a guidance to drivers passing by at different time in a day.

VI. Conclusion

In this paper, we focus on the shortest path problem in dynamic road networks. We analyze the urban road conditions, which are mainly divided into two types including congested condition and uncongested condition. The hybrid speed model is proposed with the consideration of both two conditions. Subsequently, the estimation process of link travel time is analyzed. In order to better our model, we provide a method to identify the two conditions. In the numerical experiment, the heuristic A^* algorithm is adopted to take full advantage of our model. The huge traffic data obtained by detectors are used to reflect the real situation. From the result and analysis, we can find that our speed model is adaptive and can provide the predicated travel time of the shortest path in the dynamic road networks. Besides the suggestions for drivers, some findings based on hybrid speed model and huge traffic data, such as the link travel time variance, may be useful to road designers and researchers.

Future work is needed to improve the accuracy and facticity of our model, namely the method to identify the traffic conditions.

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